**EE 488 Power Systems Analysis I**

**Problem Set #3 Solutions**

1. **(40 pts)**

A diagram of a circuit

Description automatically generated

Fig. 1. 3-bus system for Problem 1

1. **(5 pts)** Convert the system shown in Fig. 1 to per unit using the given power base and a voltage base of 13.8 kV. Redraw the circuit with p.u. quantities labeled throughout.

A diagram of a diagram

Description automatically generated

1. **(5 pts)** Find the 3x3 admittance matrix that describes the system.

Here are the particulars for computing Ybus:

1. **(5 pts)** Write out the symbolic equations needed for Gauss-Seidel solution of the power flow problem for this system. Write the two equations needed post-iteration to find the final values for P and Q at the slack bus.

Here are the equations needed to iterate for voltages at bus 2 and bus 3:

Because the system is so small, there is not much savings realized with Gauss-Seidel. The only updated voltage that can be used in the current iteration is V2(i+1) when calculating V3. After the voltage iteration is complete, the real and reactive power at the generator can be found using the following equations:

1. **(10 pts)** Write a MATLAB program to carry out the Gauss-Seidel iterations to find the unknown load bus voltages and final real and reactive power for the source.

The entire MATLAB program appears on the following page:

A screenshot of a computer program

Description automatically generated

A screenshot of a computer program

Description automatically generated

Here is the output of the MATLAB run:

A white text with black numbers

Description automatically generated

. . .

A number with black numbers

Description automatically generated with medium confidence

A black text with black text

Description automatically generated with medium confidence

1. **(5 pts)** Using a power base of 100 MVA and a voltage base of 13.8 kV, find the actual steady state values for voltage at the load buses, in polar form, as well as the final values for P and Q at the slack bus. See results above.
2. **(10 pts)** Set this system up in PowerWorld and simulate to check your final answers.

A diagram of a diagram

Description automatically generated

Here is a snip of the Buses table from PowerWorld. We can see pretty good correlation of the voltages at buses 2 and 3 with what we calculated above using MATLAB:

A screenshot of a computer

Description automatically generated

1. **(20 pts)** A 3-phase, 60 Hz, wye-connected, 13.8 kV generator is connected directly to a step-up transformer rated at 13.8 kV(Y):138 kV(Y). Both the generator and transformer are rated at 100 MVA. Series reactance of the step-up transformer is 0.15 p.u. per phase and all other transformer impedances are neglected. Table 1 provides additional specifications of the generator needed for fault analysis. For simplicity, you may assume that the generator has no internal steady state impedance, only transient reactance values.

Table 1. Generator transient response specifications for problem 2

|  |  |  |
| --- | --- | --- |
| **Name** | **Symbol** | **Value** |
| Direct axis, sub-transient reactance |  |  |
| Direct axis, transient reactance |  |  |
| Direct axis, synchronous reactance |  |  |
| Direct axis, short-circuit, sub-transient time constant |  |  |
| Direct axis, short-circuit, transient time constant |  |  |
| Armature time constant |  |  |

The high voltage side of the transformer is connected to a three-phase circuit breaker “A” and no load. The generator operates at rated voltage. At t = 0 seconds, a bolted, three-phase fault occurs immediately after the circuit breaker. This is shown in Fig. 3.

A diagram of a circuit

Description automatically generated

Fig. 3. Three phase circuit for problem 2

* 1. **(10 pts)** What is the magnitude of the sub-transient fault current that flows instantaneously through circuit breaker “A” immediately after the fault occurs (time t = 0+)? Give your answer both in per unit and in amps. Remember that the transformer reactance is in effect throughout the fault transient.

Our answer here depends on how you calculate the base current. I have multiplied the wye voltage (138 kV) by 3, but you might do things differently.

Other possible answers, depending on Ibase, are 1.969 kA and 3.411 kA

* 1. **(10 pts)** Assuming circuit breaker “A” interrupts the fault current at three cycles after fault occurrence, what is the rms value of the asymmetrical fault current through the breaker at the time of interruption? Give your answer both in per unit and in amps. *You must assume worst case DC offset.*

The following uses Table 7.2.

Other possible answers are: 2.117 kA and 3.666 kA

1. **(30 pts)** Consider an expansion of the circuit from problem 2 shown in Fig. 4.

A diagram of a circuit

Description automatically generated

Fig. 4. Expanded system for problem 3

Table 2. Bus information from steady state, pre-fault operation

A screenshot of a computer

Description automatically generated

Note that because of the synchronous load, the voltage at node 5 is a little above unity in terms of per unit. At time t = 0, a three-phase fault occurs at hypothetical bus #3. This is represented again by a switch closing to ground.

1. **(10 pts)** Using nodal analysis, determine the Thevenin equivalent voltage, per unit, at node 3 at the instant the fault occurs.

There is a subtle issue here. In problem 2, I said that you can assume the generator has no internal reactance that impacts steady state. If you assume the same for the synchronous motor, then the following circuit is to be analyzed to find V3 at the instant of the fault. If you assumed reactances for the “sources”, then that’s fine too – it will make hardly any difference.

A diagram of a voltage

Description automatically generated

We would guess that the magnitude and phase of V3 would be near the V1 and Vs midpoint:

1. **(5 pts)** Redraw the Thevenin equivalent circuit appropriate for just after the fault occurs, showing the Thevenin voltage source, VF, impedances (you can do some combining), and fault current labeled I”F.

Now we include all the reactances:

A diagram of a current

Description automatically generated

1. **(10 pts)** Calculate the magnitude of the instantaneous fault current at time t = 0+. Do your calculation in per unit first and then calculate the actual current in amps.

We will assume rms here, but if you want peak, then just multiply by .

1. **(5 pts)** Compare the current obtained for this problem with the initial fault current obtained in Problem 2. Why does this occur (in your own words)?

In problem 3, we arrived at 6.65 p.u. for the peak current. In problem 2, it was 4.71. This is because the synchronous motor has become a generator, effectively providing another power source to the circuit. This power comes from the mechanical energy stored in the motor at the time of the fault.

1. **(10 pts)** A 60 Hz, three-phase transmission line is being designed with the following characteristics, shown in Table 3.

Table 3. Proposed transmission line characteristics

|  |  |
| --- | --- |
| **Characteristic** | **Value** |
| Normal operating voltage | 161 kV |
| Maximum operating voltage | 165 kV |
| Continuous current rating | 1.8 kA |
| Maximum calculated symmetrical fault current (at max operating voltage) | 35 kA |

**(5 pts)** Note that the X/R ratio is unknown. Using Table 7.10 on p. 456 of the textbook, find a suitable outdoor circuit breaker. Be sure to support your answer. You may simply indicate which line on the table is your choice, e.g, “10th line from the bottom.”

The 161 kV class has 4 entries. Maximum voltage is okay, under the 169 kV limit. Since the X/R ratio is unknown, the max symmetrical fault current cannot exceed 35k/0.8 = 43.75 kA. Thus, we must go with 50 kA. Rate continuous current is okay. This is the 13th line from the bottom on Table 7.10 (last line in the 161 kV class).

**(5 pts)** In this voltage class, what would have to change in the design characteristics of Table 3 in order to select the next breaker up the list (with less capability)?

The main difference in going up one line on the table is the maximum symmetrical fault current at maximum voltage. In order to go to 40 kA, if we don’t know the X/R ratio, we would need no more than 32 kA for the bottom entry of Table 3 above. Obviously, if we know the X/R ratio, then we can use that to help decide.